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APPLICATION OF MATHEMATICAL MODELLING IN BEEF HERD MANAGEMENT – A REVIEW*

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Abstract

In this paper the possibilities and the advantages of the application of mathematical modelling technique to the management of beef cattle are discussed. The management of beef herd is inseparably associated with making decisions concerning such activities as replacement/culling, insemination, feeding and marketing of animals. The present survey includes models that are part of a whole-farm management strategy; the models of farm enterprise, consisting of both cow-calf and fattening segments; the models of either cow-calf or fattening segment; and the models of single animals. In spite of a considerable variety of beef models there are many similarities in their mathematical formulation. Methodologically, beef models can be classified into optimization and simulation models. The methods commonly used in constructing and solving the optimization models are linear programming and dynamic programming expanded with Markov decision processes. Dynamic programming and Markov decision processes are also very often applied to solve the stochastic simulation models. Most models adopt the dynamic approach, which is relevant considering the changes in herd composition and single animal performance over time. The necessity of taking into account random variation, typically found in animal breeding and production, causes that a large number of models are stochastic. Mathematical models of beef cattle management are used mainly as research tools and teaching aids, and still not so many of them are applied directly in supporting decisions in commercial beef herds.

Key words: mathematical modelling, optimization, simulation, herd management, beef cattle

Beef farming systems across the world show considerable differences in the economic efficiency of beef production. Profitability of beef cattle enterprises depends on many factors including biological performance of animals, management strategies, natural conditions and marketing possibilities. However, it is influenced to the highest degree by costs and returns and a key element seems to be beef price (Bruce et al., 1999 a, b; Wolfová et al., 2004). In Poland, the relatively low live weight prices and high costs of production are, according to many opinions, a significant limitation to the development of beef cattle breeding. After Poland's accession to the

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European Union in 2004, beef prices increased and their periodical fluctuations have been observed (Olszańska, 2008).

Because farmers have rather little influence on beef prices, attempts to improve the economic results of beef production through better management of the herd have been undertaken. Management comprises decision-making for the purpose of accomplishing desired goals and objectives. In order to make optimal decisions concerning a certain unit (animal, group, herd, farm), farm managers need the knowledge about the present state of the unit, the relation between the factors used and the resulting effects given the present state, personal preferences and all restraints of legal, economic, physical or personal kind. The decision process is further complicated by the fact that most of the knowledge is associated with uncertainty (Kristensen and Jørgensen, 1999).

The choice of the optimal decisions can be supported by the use of mathematical modelling. Mathematical model is a simplified representation of a system aimed at detecting the quantitative relationships between variables and predicting the effects of their changes, assuming a compromise between accuracy and tractability. The main advantages of model based decision support include the ability to take individual conditions into account, a concise framework for combination of information from different sources, direct representation of uncertainty and efficient search algorithms for determination of optimal decisions. Models may further contribute with extensive sensitivity analyses concerning optimal decisions, deviating conditions and parameter values (Jalvingh, 1992; Kristensen and Jørgensen, 1999; Pla, 2007). Advanced computational methods and computers used in the modelling process allow considering more aspects of a decision compared to the advice given by experts (e.g. veterinarians or agricultural consultants) or application of general norms, standards and recommendations. The decisions can be taken at different planning horizons: long-term (strategic), medium-term (tactical) and short-term (operational) (Jalvingh, 1992; Kristensen and Jørgensen, 1999).

Numerous investigations indicate the usefulness of mathematical modelling to support beef cattle management decisions (Table 1). The models described in literature can be classified as:

– models that are part of a whole-farm management strategy

– the models of farm enterprise, consisting of both cow-calf and fattening (e.g. feedlot) segments

– the models of either cow-calf or fattening segments

– the models of single animals

Table 1. Reviewed beef cattle models

Authors	Year	Method used	Characteristics of the model
1	2	3	4
Werth et al.	1991	Simulation	Model for evaluation of the influence of reproductive performance and management decisions on net income in beef production.
Keele et al.	1992	Simulation	Model to predict the effects of level of nutrition on composition of empty body gain in beef cattle.

Table 1 – contd.

1	2	3	4
Lamb et al.	1992	Simulation	Model for evaluation of mating systems involving five breeds in integrated cow-calf-feedlot production enterprise.
Koots and Gibson	1998	Simulation	Bio-economic herd level model to estimate the economic values for beef production traits.
Kilpatrick and Steen	1999	Simulation	Model for prediction of beef cattle growth and carcass composition.
Makulska and Kristensen	1999	Markov process	Model to optimize fattening strategy of an individual young bull and a group of bulls.
Pang et al.	1999	Simulation	Model for evaluation of the effects of calving season and weaning age on bio-economic efficiency of beef herd.
Tess and Kolstad	2000	Simulation	Model to simulate dynamic relationships among beef cattle genotypes, physiological states, forage quality and management in range environments of Montana, USA.
Nielsen and Kristensen	2002	Multi-level Markov process	Model for determination of optimal decisions in organic steer production – including winter feed level, grazing strategy and slaughtering policy.
Pihamaa and Pietola	2002	Optimization Dynamic programming	Model for determination of optimal beef cattle management under agricultural policy reforms in Finland.
Williams and Jenkins	2003	Simulation	A dynamic model of metabolizable energy utilization in growing and mature cattle.
Hoch and Agabriel	2004	Simulation	A dynamic model to estimate beef cattle growth and body composition.
Costa and Rehman	2005	Optimization Linear programming	Model for maximization of the asset value of cattle and the economic returns from beef production systems of Central Brazil.
Rotz et al.	2005	Simulation	Model for simulating feed intake, animal performance and manure excretion in beef farms.
Veysset et al.	2005	Optimization Linear programming	Model for determination of optimal combination of production activities (animal and grassland) under many constraints in France.
Wolfová et al.	2005	Simulation	Model for economic evaluation of beef bulls' utilization in a variety of production systems in the Czech Republic.
Crosson et al.	2006	Optimization Linear programming	Model for determination of the optimal beef production systems in Ireland.
Havlik et al.	2006	Optimization Linear programming	Model for agri-environmental policy analysis involving suckler cow farm production system in the White Carpathians, Czech Republic.
Makulska	2006	Optimization Dynamic programming	Model for supporting the decision processes in bull fattening.

Table 1 – contd.

1	2	3	4
Oltjen and Ahmadi	2006	Optimization Linear programming	Model for ration formulation and projection of profit or loss in a feedlot system.
Villalba et al.	2006	Simulation	Model for stochastic simulation of mountain beef cattle systems in Spanish Pyrenees.
Gradiz et al.	2007	Simulation	Model for integration of beef cow–calf production system with sugarcane production in Japan.
Reisenauer Leesburg et al.	2007	Simulation	Model for evaluation of calving seasons and marketing strategies in beef enterprises located in Northern Great Plains, USA.

Mathematical methodologies used in livestock herd modelling

Methodologically, livestock models, including beef cattle models, can be divided into optimization and simulation models (Figure 1). Optimization models allow determining optimal outcomes given the objective function of expected utility or function of profit that is maximized subject to production alternatives, prices and resources availability. Simulation models are developed mainly to improve the understanding of the systems by studying their behaviour under different conditions. They calculate the expected utility under a given set of parameters and decision rules. When simulation models are used to determine "optimal" strategies the aim is to find the optimal set of decision rules given the precision in the current knowledge of the parameters (Kristensen and Jørgensen, 1996, 1999; Pla, 2007).

Within optimization and simulation two model categories can be distinguished: deterministic and stochastic. In deterministic models the assumptions are that the real system has no random variation while in stochastic models the variation of variables and parameters is represented through appropriate probability distributions. The stochastic approach is usually more suited to solve the models of livestock management since their parameters are associated with animals utilisation and therefore have a random component.

Models can also be classified as static and dynamic. In static models time is not included as variable, so these models are not able to simulate the behaviour of the system over time and therefore are hardly relevant to livestock management problems. On the contrary, dynamic models have time as an important driving variable (Javlingh, 1992).

Linear and dynamic programming (with Markov decision processes) are the mathematical methods often used in constructing and solving the optimization models. Dynamic programming and Markov processes are also very often applied to solve the dynamic stochastic simulation models. Other methods, like decision graphs, Bayesian networks and Monte Carlo simulations method are not so frequently used for model based decision support and therefore they are not described in this paper (Kristensen and Jørgensen, 1999).

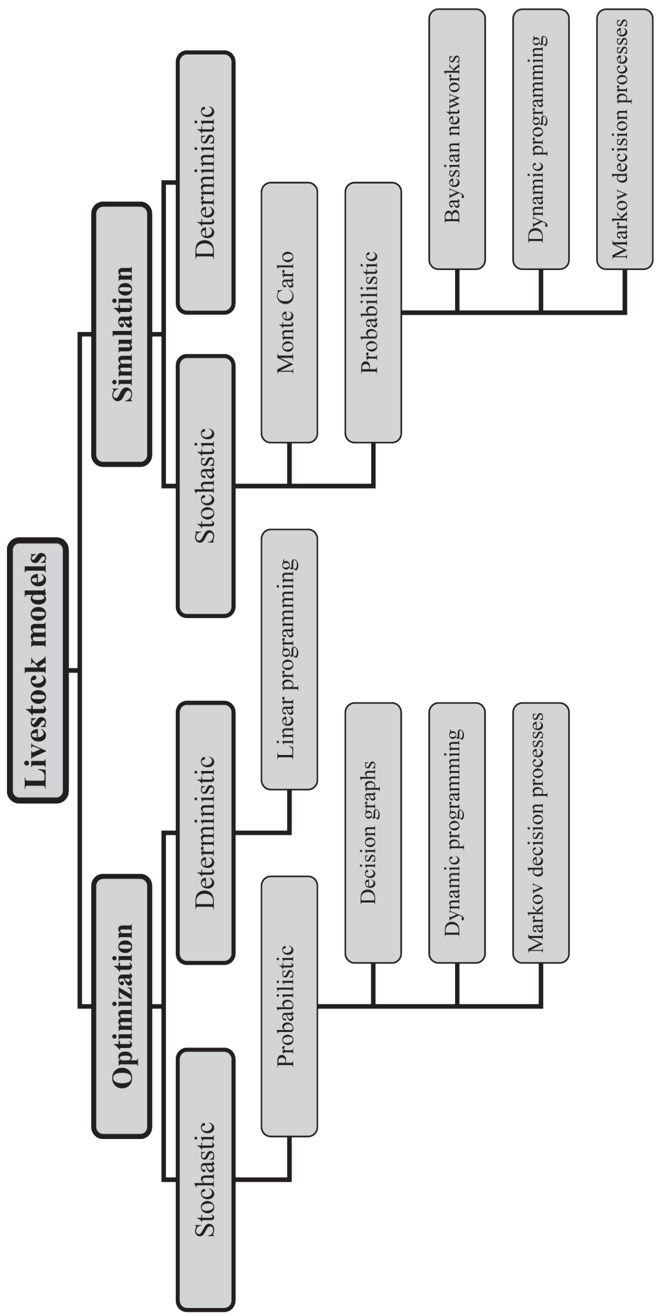

Figure 1. Methodological classification of livestock models

Linear programming (LP) is a method of mathematical programming in which the objective is to maximize or minimize linear function subject to the restrictions normally referred to as resource constraints. Four basic assumptions are essential to determine whether LP is applicable to a particular problem and whether it will provide a meaningful and precise answer (Jalvingh et al., 1997):

– additivity and linearity in input and output coefficients;
– divisibility in resources and products;
– finiteness of alternative processes and resource restrictions;
– single-valued expectations.

The method most frequently used to solve linear programming problems is the simplex algorithm. It specifies each step that is to be taken during the solution process, and is actually a trial-and-error procedure for problem solving. However, it is constructed in such a way that each trial results in an improved answer. The algorithm guarantees that, if an optimal value exists, it will be found within a finite number of steps (Heady and Chandler, 1958).

The information on the economic contribution of various resources to the measure of performance (e.g. profit) is very useful for farm managers and animal producers. The simplex method provides this information in the form of shadow prices for the respective resources. The shadow price for a given resource measures the marginal value of this resource, that is, the rate at which profit would be increased (slightly) with the increase of the amount of this resource (Jalvingh et al., 1997).

Dynamic programming (DP) has become widely accepted as one of the main tools for optimization (Bellman, 1957; Kristensen, 1987). DP is applicable to processes involving a sequence of decisions over a given, finite or infinite period of time (planning horizon) split into stages. At each stage, the state of the process is observed and a decision concerning the process has to be made. To solve finite stage decision problems the most commonly used method is value iteration. It consists in maximizing (or minimizing) a value function, representing the expected total rewards (outcomes) from the present stage until the end of the planning horizon. Optimal decisions depending on stage and state are determined backwards step by step as those maximizing (or minimizing) the value function. This way of determining an optimal policy is based on the Bellman principle of optimality: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman, 1957). For infinite stage problems the most relevant optimization technique is policy iteration. This method was introduced by Howard (1960) who combined the dynamic programming with the mathematically well-established notion of a Markov chain. The combination, characterized by sequential and stochastic approach, was named Markov decision process (MDP). Usually Markov decision processes are considered as optimization models. However, since MDP takes into account the probabilistic nature of herd, it can also be applied in dynamic stochastic simulations. Use of simulation in Markov decision processes allows comparing the consequences of different non-optimal and optimal policies (Kristensen, 1994; Pla, 2007).

Optimization models

Linear programming

Livestock management applications of linear programming (LP) are numerous (e.g. Glen, 1980; Olson et al., 1980; Costa and Rehman, 2005; Veysset et al., 2005; Crosson et al., 2006; Havlik et al. 2006; Oltjen and Ahmadi, 2006). Most often LP is used for whole-farm planning (Veyseet et al., 2005; Costa and Rehman, 2005; Crosson et al., 2006; Havlik et al., 2006) and for formulation of least-cost feeding rations (Glen, 1980; Oltjen and Ahmadi, 2006).

Some recent examples of the application of linear programming to beef cattle management are connected with the necessity of re-evaluating optimal systems in beef farms after the reform of the Common Agricultural Policy (CAP) in 2003. The models developed in Ireland (Grange Beef Model) and in France (Opt'INRA) are used to identify the best solutions in these new circumstances. The aim of the Grange Beef Model is to determine the optimal Irish beef production system based on management alternatives focusing on grazing temperate grassland, within the livestock and feeding specifications (Crosson et al., 2006). The French optimization model Opt'INRA, described in 2005 by Veysset et al. (2005), determines the optimal combination of various production activities (including animals and grassland). The objective of the Opt'INRA model is to maximize the gross margin of the farm subjected to numerous constraints (agronomic, agri-environmental, CAP, farm area, housing, animal production, etc.). By taking into account organic farming limitations and nitrogen balance Opt'INRA model was used to study the adaptation of the production system and economic consequences of the transition of a cattle suckler system to organic farming.

The important problems of a sustainable development of beef production and environment protection were also reflected in the models of Havlik et al. (2006) and Costa and Rehman (2005). Havlik et al. (2006) developed a linear model called Beef and Grassland Biodiversity Production Optimisation Model (BEGRAB_PRO.1). This model was used to analyse the organic suckler cow farms in the Protected Landscape Area of the White Carpathians, the Czech Republic. BEGRAB_PRO.1 enables accounting not only for beef but also for biodiversity production. Biodiversity production is depicted by a system of technical constraints, which represent limitations and tasks to be respected in order to produce particular environmental goods. Costa and Rehman (2005) devised the bi-criteria model aimed at maximization of the asset value of cattle and maximization of the economic returns from Brazilian beef production systems, in the situation of a rapid spread of pasture degradation. In the model the different attitudes of the farmers towards overgrazing, pasture costs and capital availability were analysed. The model was also used to test various hypotheses to explain the overgrazing behaviour.

The examples of the application of linear programming to feeding ration formulation are the model and computer program of beef cattle management (TAURUS) developed by Oltjen and Ahmadi (2006), and the model of bull fattening devised by Makulska (2006). Generally, TAURUS is intended to formulate the least-cost rations and to project a profit or loss in feedlot operations. Output of the computer program consists of six parts: cost and performance, ration composition, price ranges, nutrient analysis of the ration, equations, and nutrient analysis of feeds in the ration. The program predicts also days on feed, live weight, carcass yield, carcass quality and the digestible energy of five different feed groups used in beef cattle diet formulation. In the model of Makulska (2006) the least-cost feeding rations for all defined combinations of fattened bull body weight and daily gain were formulated. The rations provided adequate levels of both energy and protein, within the limits of dry matter intake.

Dynamic programming and Markov decision processes

Historically, first applications of the dynamic programming method to beef cattle management were found in the papers by Bonnieux (1969), Nelson (1969) and Meyer and Newett (1970). In 1972 Kennedy developed the dynamic model of beef bull fattening and marketing. The model consisted of two decision systems: a live weight sequencing system and a ration composition system (Kennedy, 1972). A similar dynamic programming problem, i.e. determination of the optimal feeding and marketing strategies for pasture-fed beef cattle, was described in the model of Clark and Kumar (1978).

The more recent research on the optimization of the bull fattening process by means of dynamic programming is the model of Pihamaa and Pietola (2002). The aim of this model was to increase the returns from Finnish beef production through the determination of the optimal feeding and time of slaughtering under alternative policy, price and forage cost scenarios. An important input to the model were subsidies which still considerably influence optimal carcass weight and farmers' income in Finland. The dynamic programming, as a tool for gaining a valuable insight into the factors that determine the efficiency of fattening process of young bulls, was also described by Makulska (2006). In her model two decision problems were taken into account: composition of feeding rations and strategy of fattening. Approaches adopted in the dynamic optimization of fattening strategy included: fattening to the assumed slaughter weight, fattening with the assumed duration, and cyclic fattening with the replacement option.

Although Kennedy (1972) has found dynamic programming to be a flexible tool for dealing with the dynamic problems of animal production he also mentioned the drawbacks of this method. The main drawback manifests itself in the possibility of handling only rather small models, with a few hundred states. This is connected with the problem referred to as the "curse of dimensionality" that can be described in the following way (Kristensen, 1994): If several variables are considered simultaneously and each variable is considered at a realistic number of levels, the state space grows to prohibitive dimensions and model becomes very large. Furthermore, in livestock management a hierarchical structure of decisions is often faced. The decisions are made not only at different levels that are mutually dependent but also at different time horizons. These aspects contribute even more to the dimensionality problem than the extension of state space mentioned above. In such situation a conside-rably increased computer memory is required. More complicated multi-state processes, involving decisions with varying time horizon, can be optimized by means of an efficient DP algorithm, i.e. the hierarchical Markov process (HMP), developed by

Kristensen (1988). HMP is defined as a series of Markov decision processes called sub-processes built together in one main Markov process (hierarchic structure of decision processes), so that each stage in the main process represents a sub-process. Markov decision programming technique has been mainly applied to solve the animal replacement problem, defined by Van Arendonk (1984) as follows: If the asset (animal) is used in production process, it is relevant to examine at regular time intervals whether the present asset (animal) should be replaced or it should be kept for an additional period.

In 1999 an attempt to use hierarchic Markov process to optimize fattening strategy of an individual bull and a group of bulls was undertaken by Makulska and Kristensen (1999). A special emphasis was put on supporting the decision when to terminate the fattening process. The optimization of fattening strategy considered various breeds (beef, dairy and crossbred bulls), two scales of production (single-animal level – small farms, and group level – large farms) and different intensities of fattening (intensive, semi-intensive, extensive).

In order to circumvent the "curse of dimensionality" in Markov decision programming models to more satisfactory extent than with just two levels (main process and sub-processes), Kristensen and Jørgensen (2000) introduced the notion of a multi-level hierarchical Markov process. The basic idea of the multi-level design is to expand stages of the main (founder) process to a so-called child process, which again may expand stages further to new child processes leading to multiple levels. For representation and solution of multi-level hierarchical Markov processes Java software system (MLHMP) has been developed by Kristensen (2003).

The example of the application of multi-level hierarchic Markov process is the model of steer management presented in the papers of Nielsen and Kristensen (2002) and Nielsen et al. (2004). This model is a four-level hierarchical Markov process with decisions defined only at three levels. The optimized decisions concern:

- grazing strategy (permanent or ryegrass/white clover pasture);
- feeding level in winter (high and low);
- finishing strategy (age 19–27 months);
- time of slaughter (age 19–30 months).

The objective is to optimize economically the organic steer production at single-animal level. In order to calculate technical and economic key figures (feed intake, body weight gain and net returns) characterizing the optimal policy a probabilistic Markov chain simulation was used (Nielsen and Kristensen, 2007).

Simulation models

Simulation models are well suited to dealing with variability and complexity of animal production. They can be divided into three categories: whole-herd models with emphasis on management strategies, physiological models of whole herds, and physiological models of single animals (Kristensen and Jørgensen, 1996). Beef cattle simulations are often simplified by disregarding variability beyond that created by the model's deterministic equations (Shafer et al., 2007). Such approach yields lower levels of simulated variability than that typically occurring in nature. Nevertheless, numerous examples of deterministic models are found in literature (Lamb et al., 1992 a, b, c, 1993; Koots and Gibson, 1998; Pang et al., 1999 a, b; Rotz et al., 2005; Wolfová et al., 2005). One of them is the Integrated Farm System Model (IFSM) developed by Rotz et al. (2005). This whole-farm simulation model is a comprehensive presentation of farm production where beef breeding constitutes one of the branches. It incorporates a beef herd sub-model with other farm components such as crop growth, harvest, storage, feeding, grazing and manure handling. IFSM enables predicting nutrient requirements, feed intake, growth rate and manure excretion for all animal groups making up a beef herd. Since the IFSM integrates many biological and physical processes in the farm production system it can be a useful tool for evaluation and comparison of the long-term performance, economics and environmental impacts of beef production system. The whole-farm approach is also used in the model of integrated beef cow-calf and sugarcane production in Tanegashima Island, Japan (Gradiz et al., 2007). This model simulates the total requirement for energy and protein, and subsequent losses of nitrogen via faeces and urine, throughout the reproduction cycle of a mature cow and the growing stages of her calf.

Deterministic simulation models were applied to evaluate biological and economic efficiency of purebreds, two-breed and three-breed rotational crossbreds, involving five cattle breeds in the USA. The following approaches were assumed: cow-calf segment (Lamb et al., 1992 a); feedlot segment (Lamb et al., 1992 b); and integrated cow-calf-feedlot system (Lamb et al., 1992 c). Moreover, Lamb et al. (1993) tried to account for genetic trends within breeds included in the simulations and to examine variability in the average carcass performance.

Other applications of deterministic simulation to model beef cattle production systems are those presented by Koots and Gibson (1998), Pang et al. (1999 a, b) and Wolfová et al. (2005). The model of Koots and Gibson (1998) derived economic values for genetic improvement of multiple traits in the integrated beef enterprise. Modelling a complete beef production system (as opposed to cow-calf and feedlot segments separately) was necessary to reflect a situation where market signals flow down to those making the breeding decisions. That is, although payment is based on carcass value, animals must flow through both cow-calf and feedlot segments. The model of Pang et al. (1999 a, b), called Alberta Beef Production Simulation System (ABPSS), also described the situation of complete beef production. It was composed of herd inventory, nutrient requirements, forage production and economic sub-models. The herd inventory sub-model was used to evaluate population dynamics and feed requirements in the herd. The nutrient requirements sub-model simulated nutrients and feed requirements for calves and cows depending on their physiological status (maintenance, growth, lactation and gestation) and the climatic conditions. The use of forage production sub-model allowed predicting forage growth rate, cattle grazing rate, available forage biomass, and total hectares required for grazing. The economic sub-model measured bio-economic efficiency, as net return per cow, by subtracting total cost from total return. The ABPSS model was also applied to simulate the influence of calving season and weaning age on the bio-economic efficiency of beef production systems (Pang et al., 1999 a). Wolfová et al. (2005) developed the bio-economic deterministic simulation model to evaluate the utilization of bulls in a variety of production systems in the Czech Republic. The model can simulate life-cycle production of beef cow herds with and without integrated feedlot system. It can be a valuable tool for the optimization of mating, culling and other management and marketing strategies in various beef production systems. To simulate herd dynamics the Markov chain approach was adopted. The herd was described in terms of animal's states and possible transitions among particular states at different stages. The model algorithms served as a basis to write the computer program ECOWEIGHT. This program was used to estimate marginal economic values for 16 traits in four different management systems of beef bulls in the Czech Republic. ECOWEIGHT was also applied by Krupa et al. (2005) to calculate the economic weights for production and functional traits of Slovakian Simmental cattle under alternative marketing strategies.

In order to simulate more realistic levels of variability occurring in animal production systems and in life of single animals, a variability created by the deterministic equations of the model should be supplemented by that stochastically generated (Shafer et al., 2007). Both deterministic and stochastic simulation were used in the model of Werth et al. (1991). The model was devised to evaluate how reproductive performance interacts with management practices to influence net income in a cow-calf operation for one year of production. The stochastic dynamic model was applied to simulate the reproduction performance of the cow-herd. Outputs from the stochastic model were used as inputs into the deterministic cow-herd economic simulation model that calculated the net income.

A dynamic stochastic model for simulating mountain beef cattle systems in the Spanish Pyrenees was described by Villalba et al. (2006). The model was used to assess four feeding strategies during the winter period in the conditions of autumn cal-ving. Special attention was devoted to the evaluation of the relationship between nutrition and reproductive performance of cows, considering the information about production and reproduction variability of the studied groups of animals. Tess and Kolstad (2000 a) developed a stochastic model aimed at simulation of the dynamic relationships among beef cattle genotypes, physiological states, forage quality and management in range environments of Montana (USA). Forage intake, energy and protein metabolism, growth, reproduction, lactation and differences in chemical body composition were simulated for individual animals over complete life cycles. The model was applied to evaluate the response of the production and marketing system to the changes in breeding and management strategies (Tess and Kolstad, 2000 b). In 2007 a similar bio-economic computer model of cow-calf enterprise was sed by Reisenauer Leesburg et al. (2007 a, b) to assess various calving seasons and different calf marketing strategies under conditions of the Northern Great Plains (USA). The simulated ranch utilized a rotational breeding system based on Hereford and Angus.

The simulation methods were also applied to devise the models of growth and metabolism of beef cattle. An example is the model described in the papers of Keele et al. (1992) and Williams et al. (1992 a, b), developed to predict composition of empty body gain of several breeds of beef cattle fed at different levels of nutrition. Also, Kilpatrick and Steen (1999) simulated the influence of the feeding regime (either silage only or supplemented with concentrates) on beef cattle growth and carcass composition. Their model provided information on the most economic level of concentrate feeding to achieve the animal growth and quality of carcass composition required. Williams and Jenkins (2003) used the simulation model to predict heat production attributable to maintenance and support metabolism in growing and mature purebred and crossbred cattle. Hoch and Agabriel (2004 a, b) designed a mechanistic dynamic model in which beef cattle growth and body composition were simulated for different animal types (sex, breed) under various nutritional conditions. Mechanistic modelling does not require much data for model development but it assumes basic understanding of the process. The equations in the mechanistic model are derived from some theory or hypothesis about the fundamental nature of the process. This is in contrast to other quantitative models, which use the equations derived from observations in the real world, but not necessarily representing any understanding of the casual mechanism at work (Morris, 2006). The model of Hoch and Agabriel (2004 a, b) was constructed on the basis of variations of body protein and lipid contents. Proteins and lipids in carcass and non-carcass tissues were distinguished to account for different energy metabolism of these two components of the body. Evolution of each compartment was determined by the instantaneous balance between synthesis and degradation, which depends on the physiological age of the animals and on metabolizable energy supply. Empty and full body weights were deduced from protein and lipid contents through allometric equations which in biology are used to describe the morphological evolution of species, and are based on the relation between an organism's size and the size of any part of the organism (Warriss, 2000).

In conclusion, mathematical modelling can be applied to solve complex decision problems appearing in livestock management. Most often decisions concern feeding, insemination, marketing, culling/replacement of animals.

Beef cattle models display many similarities in their mathematical formulation despite a considerable variety in relation to the undertaken problem. Methods that are commonly employed in modelling are optimization and simulation. Many models adopt the dynamic approach, which is relevant considering the changes in herd composition and single animal performance over time. The necessity of taking into account a random variation, typically occurring in animal breeding and production, causes that a large number of models are stochastic.

Nowadays the mathematical models of beef cattle management are used mainly as research tools and teaching aids. Unfortunately, still not so many models are applied directly in supporting decisions in commercial herds. In order to create an efficient decision support system for beef cattle, further development of the adequate methodology maximizing the farmer's utility is necessary. Since various methods have different properties a main challenge will probably be to combine some methods. Taking into consideration more and more variables and parameters results in the present models becoming very large. Therefore, a high priority should be given to the circumvention of the dimensionality problems. Another important issue is data acquisition and their transformation for filtering and organization of databases. The scarcity of real farm data often significantly hinders the estimation of model parameters at the herd level and the external validation of the devised model. An extensive research on the methodology observed in last years is not accompanied by a simultaneous development of software tools and the increase of farmers' experience in computer handling. Hence, a more widespread use of model-based decision support methods at the farm level demands a close cooperation between researchers, and advisory and training services. Equally important is the accessibility to powerful personal computers and the development of comprehensible software well fitted to the needs of herd managers.

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Zastosowanie modelowania matematycznego w zarządzaniu stadem bydła mięsnego – artykuł przeglądowy

STRESZCZENIE

W pracy przedstawiono możliwości zastosowania modelowania matematycznego do wspomagania decyzji w zarządzaniu stadem bydła mięsnego. Zaprezentowano problemy decyzyjne dotyczące m. in. żywienia, inseminacji, brakowania/zastępowania użytkowanego zwierzęcia kolejnym osobnikiem i sprzedaży/uboju zwierząt. Uwzględniono modele zarządzania stadem bydła mięsnego będące częścią ogólnej strategii zarządzania fermą, modele ferm złożonych z dwóch segmentów: stada krów mamek oraz opasanego młodego bydła, modele jednego z powyższych segmentów, jak również modele wzrostu zwierzęcia i zmian składu tuszy. Mimo znacznej różnorodności modeli stosowanych w zarządzaniu stadami bydła mięsnego istnieje wiele podobieństw w ich matematycznym formułowaniu. Ze względu na zastosowaną metodykę modele można podzielić na optymalizacyjne i symulacyjne. Do konstruowania i rozwiązywania modeli optymalizacyjnych najczęściej wykorzystuje się programowanie liniowe i dynamiczne z rozszerzeniem o tzw. procesy decyzyjne Markova. Programowanie dynamiczne i procesy decyzyjne Markova stosowane są także do rozwiązywania dynamicznych stochastycznych modeli symulacyjnych. Konieczność uwzględnienia czynnika czasu i zmienności losowej występującej w hodowli i użytkowaniu zwierząt sprawia, że większość modeli ma charakter dynamiczny i stochastyczny. Obecnie matematyczne modele zarządzania stadami bydła mięsnego używane są głównie jako narzędzia badawcze i pomoce dydaktyczne. Szersze ich wykorzystanie do wspomagania decyzji podejmowanych przez farmerów wymaga ścisłej współpracy naukowców, służb doradczych i szkoleniowych, dalszego rozwoju komputeryzacji na terenach wiejskich oraz większej dostępności do odpowiedniego oprogramowania.